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Job Design and Randomization in Principal Agent Models

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Summary: We analyze task allocation and randomization in Principal Agent models. We identify a new rationale that determines the allocation of tasks and show that it can be optimal to assign tasks that are very different to one agent. Similar to randomization, the reason to assign several tasks to one agent is to mitigate the effect of the participation constraint. We show that the allocation of tasks can be used as a substitute if randomization is not feasible.

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1 Introduction

Two important questions for the organization of firms are the design of incentive contracts and the allocation of tasks. We analyze two seemingly unrelated contractual regimes in principal agent relationships: ex-ante randomization in incentive contracts and the allocation of tasks. It is well known that randomization over simple contracts can be optimal in the second-best (e.g., Fellingham et.al. 1984, Arnott and Stiglitz, 1988). However, we rarely observe contracts that include randomization and randomization is usually regarded as a technical result with little relevance for real-world incentive contracts. On the other hand, contracts regularly specify some allocation of tasks. Starting with Holmstrom and Milgrom (1991), an extensive literature on multi-task agency analyzes the optimal allocation of tasks. This literature focuses on effort substitution and argues that only one task should be allocated to an agent (or, more general, that tasks should be homogeneous with respect to ease of performance measurement).

We identify a new rationale that determines the optimal allocation of tasks: the effect of the outside option. We analyze randomization in a simple model with one task and then develop a multi-task model where the principal allocates working time across different tasks. Randomization over wage schedules and the allocation of different tasks to one agent are similar in the sense that the only reason to randomize or to allocate two tasks to an agent is to mitigate the effect of the outside option. In most contractual relationships, randomization is not feasible because parties do not have access to a verifiable randomization device. We show that the allocation of two tasks to one agent can serve as a substitute for randomization.

Our result that it can be optimal to assign two tasks that are very different is the opposite of the conclusions of the multi-task literature. The reason for the different results is that most studies of multi-task agency use the linear model of Holmstrom and Milgrom. The linear model is special in the sense that the outside option does not affect the second-best contract except for a transfer. Hence in the linear model, effort substitution is the only factor that determines the optimal allocation of tasks.

The paper is organized as follows. In section 2 we analyze randomization if there exists only one task. In section 3 we analyze the allocation of tasks, relate our results to the multi-task literature and show that the allocation of tasks can serve as a substitute for randomization. Section 4 concludes.

2 Ex-ante Randomization

The literature distinguishes between two types of randomization. Under ex-post randomization, the wage depends on output and on a signal that is realized after the agent chooses an action where the signal contains no information about the action that the agent has chosen. Under ex-ante randomization, the contract specifies a lottery over wage functions that map output into wages. Before the agent chooses an action, a signal determines which wage function is selected. Of course, this signal cannot contain information about the action that the agent has not yet chosen. We study ex-ante randomization but we also allow for contracts that include ex-post randomization.

Consider a standard principal-agent problem. The principal's payoff $v(\pi, w) = \pi - w$ depends on output π and wages w. The agent chooses an action $a \in A$. Output is a stochastic function of a. Let $F(.|a)$ be the conditional distribution function of output with $\int \pi dF(\pi|a) < \infty \forall a \in A$. The agent's utility is $u(w, a)$ with $u'_w > 0$ and $u''_w < 0$ $\forall a \in A$. Let u^b be the supremum of u with $u^b < \infty$. If the agent rejects the contract, he receives outside utility $\overline{u} < u^b$. The agent is possibly protected by limited liability, i.e., $w \geq l$ with $l \geq -\infty$. There exists an independent random variable $\widetilde{\mu}$ with generic realization μ and c.d.f. G where μ is realized after the agent chooses a. Output and the realization of $\widetilde{\mu}$ are verifiable while a is unobservable. If the wage depends in a non-trivial way on $\widetilde{\mu}$, the contract includes ex-post randomization. A contract without ex-ante randomization consists of a measurable wage function w that maps output and realizations of $\widetilde{\mu}$

into R. An ex-ante random contract specifies a set of wage functions and a probability distribution over this set of wage functions. From now on, non-random contract refers to a contract without ex-ante randomization.

Suppose ex-ante randomization is not feasible. Consider the second-best problem for arbitrary, finite outside utilities:

$$\max_{w,a} \int \int v(\pi, w(\pi, \mu)) dF(\pi|a) dG(\mu) \tag{1}$$

subject to:

$$a = \arg\max_{a' \in A} \int \int u(w(\pi, \mu), a') dF(\pi|a') dG(\mu) \tag{IC}$$

$$\int \int u(w(\pi, \mu), a) dF(\pi|a) dG(\mu) \geq k \tag{PC}$$

$$w(\pi, \mu) \geq l \ \forall \pi, \mu \tag{L}$$

Let w_k, a_k be the solution to (1) subject to (IC), (PC), and (L) for $-\infty < k < u^b$. There exists an extensive literature that discusses various topological restrictions on payoffs, actions and the stochastic relation between action and output that guarantee the existence of a second-best contract (see Page, 1997, and literature cited therein). Since the focus of this paper is not on existence but on randomization and job design, we assume that the non-random second-best problem has a unique solution in the sense that a_k is unique and that w_k is unique except for a set of outputs that is realized with probability zero.

Assumption 1: For all $k \in (-\infty, u^b)$ exist w_k, a_k where a_k is unique and w_k is unique except for some set P of outputs with $\int_P dF(\pi|a_k) = 0$.

If ex-ante randomization is not feasible, the principal offers the contract $w_{\overline{u}}$. Let r_k denote the rent under a non-random second-best contract with $r_k = \int \int u(w_k(\pi, \mu), a_k) dF(\pi|a_k, \mu) dG(\mu) - \overline{u}$. Second-best rents can be positive for two reasons: limited liability and non-separability of the utility function.

An ex-ante random contract is binary if it selects only two wage functions with positive probability. Arnott and Stiglitz (1988) show that the principal cannot gain if the contract randomizes over more than two wage functions. Proposition 1 gives conditions for ex-ante randomization, shows that a second-best contract exists, and that second-best random contracts randomize only over contracts that are the solution to the principal-agent problem when randomization is not feasible and outside utility is some $k \in (-\infty, u^b)$. Assumption 1 does not guarantee that the second-best random contract is unique. Proposition 1 shows that it is sufficient to consider binary second-best contracts because every second-best random contract can be written as a probability distribution over binary second-best contracts.

Proposition 1 *(i) Consider $r_{\overline{u}} > 0$. If there exists $k \in (-\infty, \overline{u})$ such that $a_k \neq a_{\overline{u}}$, then ex-ante randomization is optimal.*

(ii) If ex-ante randomization is optimal, there exists a second-best contract that is binary and specifies two wage functions w_{k_1}, w_{k_2} with $k_1 < \overline{u}$ and $k_2 \geq \overline{u} + r_{\overline{u}}$ and probabilities $\frac{r_{k_2}}{r_{k_1} - r_{k_2}}$ and $\frac{-r_{k_1}}{r_{k_1} - r_{k_2}}$ that w_{k_1} and w_{k_2} are selected. The agent earns zero rent. The set of second-best contracts is the set of probability distributions over binary second-best contracts.

(iii) If $a_{\overline{u}} = a_k \ \forall k$, the non-random second-best contract $w_{\overline{u}}$ is optimal.

Proof. (i) Let $E\left[v(\pi, w_k)\right] = \int \int v(\pi, w_k(\pi, \mu))dF(\pi|a_k, \mu)dG(\mu)$. If there exists $k \in (-\infty, \overline{u})$ with $a_k \neq a_{\overline{u}}$, then Assumption 1 implies that $E\left[v(\pi, w_k)\right] > E\left[v(\pi, w_{\overline{u}})\right]$ and that $-\infty < r_k < 0$. Let $\lambda = \frac{-r_k}{r_{\overline{u}} - r_k}$. Consider a contract where $w_{\overline{u}}$ and w_k are chosen with probability λ and $1 - \lambda$. From the construction of λ it follows that the agent accepts. If ex-ante randomization is not feasible, the principal's payoff is $E\left[v(\pi, w_{\overline{u}})\right]$. Hence the principal's payoff is higher under the random contract.

(ii) Let W be the set of all w_k for $k \in (-\infty, u^b)$. The optimal contract never specifies a $w \notin W$. To see why, let $\widehat{r}$ be the rent for some $w \notin W$. Note that $\widehat{r} \in (-\infty, u^b - \overline{u})$. Then the principal could instead offer $w_{\widehat{r}}$ and increase her payoff while the rent does not decrease. Since u is bounded from above with $u''_w < 0$ and $v(\pi, w) = \pi - w$, the optimal contract is not the limit where the rent for one of the wage functions over which the contract randomizes approaches $-\infty - \overline{u}$ or $u^b - \overline{u}$.

Since (IC) and (L) have to be satisfied for every w over which the contract randomizes, the only reason for ex-ante randomization is to mitigate the effect of (PC). Since the objective function and (PC) are linear in probabilities, the principal cannot gain if the contract randomizes over more than two wage functions. Let w_{k_1},w_{k_2} denote the wage functions over which the contract randomizes. Wlog. $k_1 \leq k_2$. Since $E\left[v(\pi, w_k)\right]$ is non-increasing in k, ex-ante randomization can only be profitable if $E\left[v(\pi, w_{k_1})\right] > E\left[v(\pi, w_{\overline{u}})\right]$. Hence $k_1 < \overline{u}$ and $r_{k_1} < 0$. Since the expected rent has to be non-negative, $r_{k_2} > 0$. If ex-ante randomization is not feasible, the principal offers $w_{\overline{u}}$ with rent $r_{\overline{u}}$. Hence $r_{k_2} \geq r_{\overline{u}}$ and, therefore, $k_2 \geq \overline{u} + r_{\overline{u}}$. Since the random contract has to satisfy (PC), $E\left[v(\pi, w_{k_1})\right] > E\left[v(\pi, w_{k_2})\right]$ implies that the probabilities that w_{k_1},w_{k_2} are selected are such that the expected rent is zero. Hence, if randomization is indeed profitable, the problem of the principal is:

$$\max_{w_{k_1}, w_{k_2}} \frac{r_{k_2}}{r_{k_2} - r_{k_1}} E\left[v(\pi, w_{k_1})\right] + \frac{-r_{k_1}}{r_{k_2} - r_{k_1}} E\left[v(\pi, w_{k_2})\right] \tag{A1}$$

It remains to show that there exists a binary second-best contract, i.e., that (A1) has a solution. For all $w_k \in W$ is the expected wage greater than $-\infty$. Together with $\int \pi dF(\pi|a) < \infty \forall a \in A$ this implies that $E\left[v(\pi, w_k)\right]$ is bounded from above. Note that $E\left[v(\pi, w_k)\right]$ is continuous from the left and non-increasing in k. Since u is bounded from above, r_k is bounded from above. Note that r_k is non-decreasing in k. If r_k is continuous from the right we are done. To see that a binary second-best contract exists even if r_k is not continuous from the right, recall that $r_k \geq k - \overline{u}$. Suppose there exists $\widehat{k}$ such that $\lim\limits_{k \to \widehat{k}^-} r_k < \lim\limits_{k \to \widehat{k}^+} r_k$. Then there exists $\epsilon > 0$ such that $\lim\limits_{k \to \widehat{k}^+} r_k = \widehat{k} + \epsilon - \overline{u}$. Since $E\left[v(\pi, w_k)\right]$ is non-increasing in k, $E\left[v(\pi, w_{k+\widetilde{\epsilon}})\right]$ is constant for all $0 < \widetilde{\epsilon} \leq \epsilon$. From Assumption 1 it follows that the wage function and, therefore, $r_{\widehat{k}+\widetilde{\epsilon}}$ are constant for all $0 < \widetilde{\epsilon} \leq \epsilon$. Hence, if ex-ante randomization is optimal, (A1) has a solution and there exists at least one binary second-best contract.

If there exists more than one binary second-best contract, then any probability distribution over binary second-best contracts yields the same expected payoff for the principal. Hence any probability distribution over binary second-best contracts is a second-best contract. The argument above that expected rents under a binary second-best contract are zero also applies to second-best contracts that randomize over more than two wage functions. Hence any second-best random contract can be written as a probability distribution over binary contracts for which the expected rent is zero. Second-best implies that every binary contract that is chosen with positive probability maximizes the payoff of the principal which is true only for binary second-best contracts.

(iii) As shown in (ii), the optimal contract never randomizes over a $w \notin \cup w_k$ and, therefore, never implements an $a \notin \cup a_k$. Randomization over two w that both implement $a_{\overline{u}}$ amounts to a lottery over wages. Since $u''_w < 0$ the principal cannot gain by randomization if $a_{\overline{u}} = a_k \ \forall k$. ■

In general, the question whether ex-ante randomization is profitable depends on the feasibility of ex-post randomization. However, the conditions in Proposition 1 do not depend on whether ex-post randomization is feasible. Under ex-ante randomization, the uncertainty is resolved before the agent chooses an action but after he signs the contract. Hence, the participation constraint has only to be satisfied in expectation and ex-ante randomization can be desirable to mitigate the effect of the participation constraint. Ex-ante randomization cannot be used to mitigate the incentive constraint since for each wage function the corresponding incentive constraint has to be satisfied. By contrast, ex-post randomization can be used to mitigate the effect of the incentive constraint if the risk-aversion of the agent depends on his action. Under ex-post randomization, the uncertainty is resolved after the agent chooses an action. Hence, the incentive constraint does not have to be satisfied for every wage function over which the contract randomizes but only in expectation over wage functions.

3 Job design

The multi-task literature emphasizes that each agent should work on one task. On the other hand, many employment contracts specify explicitly or implicitly that employees spend a certain amount of their working time on one task and the rest on another task. For example, it is common that consultants spend part of their time with the client (acquiring information about the needs/problems of the client, etc.) and part of their time in the office (doing research, writing reports, etc.). This type of contract specifies wages and an allocation of time across tasks but makes no use of randomization. We refer to such a contract as job design. To see why the assignment of two tasks can be profitable, consider tasks i, j. Under job design, a contract specifies an allocation of time across tasks and, for each task, a wage function. To make tasks as heterogeneous as possible, suppose that the action for task i is verifiable while the moral hazard problem for task j is severe in the sense that an agent earns a rent in the second-best if he works only on task j, i.e., $r_{\overline{u}} > 0$. Assigning both tasks can be profitable if the rent from task j allows to reduce the wage for task i while still satisfying the participation constraint.

Dynamic principal-agent problems are considerably more complex than the static problem. We are interested in job design as a way to mitigate the effect of the participation constraint. We make two assumptions to prevent that other factors drive the allocation of time across tasks. First, we assume that principal and agent have instantaneous payoff functions which depend on current output, wages, and actions. Second, we restrict the class of contracts such that the wage depends only on the current output and on the realization of $\widetilde{\mu}$. Additionally, we assume that ex-ante randomization is not feasible. These assumptions ensure that the allocation of time across tasks is not driven by the savings behavior of the agent, by the production technology, or the fact that the principal gets a more precise signal of the action if the agent works longer on a task.

To analyze job design in multi-task situations consider the model from section 2 with n tasks. Let $A_1, A_2, ..A_n$ be a partition of A where A_i is the action set of task i. Since we are interested in situations where the principal allocates time across tasks, we assume that the principal can specify for each task i a time interval during which the agent is restricted to choose a from A_i. Let u and v be instantaneous payoff functions and let π be a flow variable where the distribution of $\pi(t)$ depends on the action in time t. Total time is normalized to one. A job design contract consists of two parts. The first part specifies for each task i a measurable wage function w_i that maps current output and the realization of $\widetilde{\mu}$ into R. The second part specifies an allocation T of time across tasks with $T : [0,1] \to \{A_1, A_2, ..A_n\}$. To simplify notation, we sometimes write wages as a function of T instead of the task with $w_{T(t)} = w_i$ if $T(t) = A_i$. If the agent accepts the contract, he chooses at every $t \in [0,1]$ some $h_t : [t,1] \to A$ subject to $h_t(\widetilde{t}) \in A_i$ if $T(\widetilde{t}) = A_i$ to maximize his expected future payoff $\int \int_t^1 \int u(w_{T(\widetilde{t})}(\pi, \mu), h_t(\widetilde{t}))dF(\pi|h_t(\widetilde{t}))d\widetilde{t}dG(\mu)$. Note that the optimal h_t does not depend on earlier wages and actions. Hence we can assume wlog. that the agent chooses only once a function $h : [0,1] \to A$ subject to $h(t) \in A_i$ if $T(t) = A_i$ to maximize his expected payoff.

The principal solves the following job design problem:

$$\max_{(T,(w_i)_{i=1}^N,h)} \int \int_0^1 \int v(\pi, w_{T(t)}(\pi, \mu))dF(\pi|h(t))dtdG(\mu)$$

subject to (L) and

$$\begin{aligned} h(.) &= \arg\max \int \int_0^1 \int u(w_{T(t)}(\pi, \mu), h'(t))dF(\pi|h'(t))dtdG(\mu) \\ h(t) &\in A_i \text{ if } T(t) = A_i \end{aligned}$$

$$\int \int_0^1 \int u(w_{T(t)}(\pi, \mu), h(t))dF(\pi|h(t))dtdG(\mu) \geq \overline{u} \tag{PC'}$$

The solution of a job design problem is a triple $(T, (w_i)_{i=1}^N, h)$.

Similar to section 2, we are not interested in topological restrictions on payoffs, actions and the stochastic relation between action and output that guarantee the existence of a second-best contract. Therefore, we assume that the principal-agent problem has a unique solution if the agent can work only on one task. If the agent works only on one task, the principal offers a contract that specifies a task and a wage function w. Let w_k^s, h_k^s be the solution to the second-best problem if the agent can only work on a single task and his outside utility is $k \in (-\infty, u^b)$. Note that h_k^s implicitly defines the task that the principal specifies and that the agent cannot be strictly better off if he chooses different actions at different points in time.

Assumption 2: For all $k \in (-\infty, u^b)$ exist w_k^s, a_k^s where w_k^s is unique except for some set of outputs P with $\int_P dF(\pi|a_k^s) = 0$ and $h_k^s(t) = a_k^s$ except for some $t \in T$ with $\int_T dt = 0$.

Let r_k^s be the rent if the agent can only work on one task with $r_k^s = \int\int u(w_k^s(\pi, \mu), a_k^s) dF(\pi|a_k^s) dG(\mu) - \overline{u}$. Note that payoff functions and the class of contracts are deliberately chosen such that the optimal allocation of time across tasks is not influenced by the production technology, the savings behavior of the agent, or the objective to receive a more precise signal about the action.

A job design contract is binary if it allocates a positive amount of time to two tasks. Proposition 2 gives conditions such that a non-trivial allocation of time across tasks is optimal and shows that second-best job design contracts can be written as an allocation of time across contracts that are optimal if the agent can work only on one task and choose his action only once. If the second-best job design contract is not unique, then the set of all second-best job design contracts is the set of time allocations over binary second-best contracts.

Wlog. let $a_{\overline{u}}^s \in A_1$.

Proposition 2 *(i) Consider $r_{\overline{u}}^s > 0$. If there exists $k \in (-\infty, \overline{u})$ such that $a_k^s \notin A_1$, there exists a second-best job design contract that is binary and that allocates a positive amount of time to two tasks i, j with wages $w_i = w_{k_i}^s$, $w_j = w_{k_j}^s$ and $k_i < \overline{u}$, $k_j \geq \overline{u} + r_{\overline{u}}^s$. The agent earns zero rent.*

(ii) The set of second-best job design contracts is the set of time allocations over binary second-best job design contracts.

(iii) If $a_{\overline{u}}^s = a_k^s\ \forall k$, under the optimal job design contract the agent works only on task one, i.e., $T(t) = A_1 \forall t \in [0,1]$ and $w_1 = w_{\overline{u}}^s$.

Proof. Let W^s be the set of all w_k^s for $k \in (-\infty, u^b)$. For a given job design contract let $T_i = \{t | T(t) = A_i\}$. If the agent accepts the contract, he chooses h to maximize $\int \int_0^1 \int u(w_{T(t)}(\pi, \mu), h(t)) dF(\pi | h(t)) dt dG(\mu)$ subject to $h(t) \in A_i$ if $t \in T_i$. Hence, the optimal $h(t)$ depends only on $w_{T(t)}$. Hence, the expected payoff of the principal from task i is independent from the wages that are offered for other tasks. Therefore, $w_i \in W^s$ if $m(T_i) > 0$ where m denotes the Lebesgue measure. From Assumption 2 it follows that $h(t)$ is constant for all $t \in T_i$. Hence the expected payoff of the principal from task i is linear in the time that is allocated to task i. Note that the exact time allocation does not matter, because payoffs depend only on how long the agent works on a task. Since the incentive constraint and (L) have to be satisfied in every t, the only reason to allocate time to more than one task is to mitigate the effect of (PC'). Since the objective function and (PC') are linear in $m(T_i)$, the principal cannot gain if she allocates time to more than two tasks. The remainder of the proof is similar to the proof of Proposition 1. ■

The multi-task literature (e.g., Holmstrom and Milgrom, 1991) analyzes how incentives for one task distort the agent's allocation of effort across tasks. This effect is absent in the model above. The reason is not that the principal allocates time across tasks (loosely speaking, in Holmstrom and Milgrom, the principal does not allocate time but the agent works on all tasks simultaneously) but the assumption that payoffs are separable across tasks. Of course, if the agent's disutility from an action is not independent across tasks, assigning two tasks can have a positive or negative effect. Whether the positive effect from the mitigation of the participation constraint or the effect from effort substitution dominates depends on the specification of the model. Our result that it can be optimal to assign two tasks that are very different is the exact opposite of the conclusions of the multi-task literature. For tractability reasons, nearly all papers that analyze multi-task situations use the linear model of Holmstrom and Milgrom, 1991. Linear models are special because the assumption that the agent has CARA utility implies that the outside option does not affect the second-best contract except for a transfer, i.e., $a_{\overline{u}} = a_k \ \forall k$ (and $a_{\overline{u}}^s = a_k^s \ \forall k$ if there are multiple tasks). Therefore, there is no way how randomization or job design can mitigate the participation constraint (Proposition 1(iii) and Proposition 2(iii)). Hence, the conclusion of the multi-task literature that it is never optimal to assign two tasks (or that similar tasks should be grouped together) does not necessarily hold for different agency models.

In the real world, randomization is usually not feasible. On the other hand, there exist many contracts which specify an allocation of tasks. Our results imply that the allocation of tasks can serve as a substitute for randomization. Consider the example from the beginning of this section where there exist two tasks i, j. Suppose that each task requires one unit of time and that there are two agents, each endowed with one unit of time. For simplicity, assume that effort on one task has no effect on the other task and that there is no cooperation or competition among agents. If randomization is not feasible and every agent works on one task, then one agent works on task j with wage $w_{\overline{u}}$ and earns a net-rent $r_{\overline{u}} > 0$. The other agent works on task i and his net-rent is zero. If both agents spend half of their time on task j with wage $w_{\overline{u}}$, they each earn a net-rent $0.5r_{\overline{u}}$ on this task. Since $r_{\overline{u}} > 0$ and since there is no incentive problem on task i, the principal can reduce the wage for task i relative to what he has to pay if one agent works only on task i and still satisfy the participation constraint. Hence if randomization is not feasible, it is optimal to ask both agents to work on both tasks.

We have shown in section 2 that the outside option is the only factor that determines whether randomization is optimal. On the other hand, there are many factors which can potentially affect the optimal allocation of tasks. To simplify the exposition, we concentrate on the effect of the outside option. The reason why the allocation of tasks can be used as a substitute for randomization is that their effects with respect to the outside option are similar. Intuitively, since expected payoffs are linear in probabilities, a contract that calls for working Monday to Thursday on task i and Friday on task j is similar to a contract that assigns task i with probability 0.8 and task j with probability 0.2. Hence randomization and job design both allow to implement a convex combination of either two simple contracts (i.e., randomization) or of two contracts that specify only one task (job design).

Proposition 2 shows that the allocation of two tasks to one agent can mitigate the effect of the participation constraint in the same way as ex-ante randomization. The logic that randomization and job design can be used to mitigate the effect of the participation constraint also applies in the first-best. In the first-best, the principal maximizes her payoff subject to (PC) and (L). Hence the only reason why randomization and job design can be profitable in the first-best is to mitigate the effect of the participation constraint. Similar to the second-best contracts described in Proposition 1 and 2, random first-best contracts randomize only over actions that are implemented in the first-best for some outside utility k when randomization is not feasible and first-best job design contracts allocate a positive amount of time only to actions and tasks that are implemented in the first-best for some outside utility k when the agent can work only on one task.

The multi-task model in section 3 is special since it concentrates on the effect of the participation constraint but does not consider effort substitution. Besides its tractability, the advantage of the model is that all second and first-best contracts consist of probability distributions respective time allocations across contracts that are optimal if randomization is not feasible respective if the agent can work only on one task. This allows us to use the results from the extensive literature on principal-agent models with no randomization and one task to analyze contracts in the more complex case when randomization is feasible or when there are several tasks.

4 Conclusion

Ex-ante randomization and the allocation of tasks are two seemingly unrelated contractual regimes that are usually analyzed separately. This paper uses a simple framework to analyze both ex-ante randomization and the allocation of tasks and to show the similarities between the contractual environments. Most of the multi-task literature focuses on effort substitution and argues that only one task should be allocated to an agent (or, more general, that tasks should be homogeneous with respect to ease of performance measurement). We identify a new rationale that affects the optimal allocation of tasks and show that it can be optimal that the agent works on two tasks - even if they are very different. Assigning two tasks to one agent can be optimal if it mitigates the effect of the outside option. Similar, the effect of the outside option is the reason why ex-ante randomization can be profitable. While we rarely observe contracts that include randomization, there exist many contracts which specify the allocation of tasks. The paper shows that the allocation of tasks can serve as a substitute for randomization if randomization is not feasible.

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